Pearson Edexcel

Mark Scheme (Results)

January 2021

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2
Paper WFM02/01

Question Number	Scheme	Marks
1.	$\mathrm{i}\left(1+\sqrt{3}\right)=\dfrac{\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}}{\mathrm{i}^2\left(1+\sqrt{3}\right)+3}$	M1
	$-\mathrm{i}\left(1+\sqrt{3}\right)^2+3\mathrm{i}\left(1+\sqrt{3}\right)=\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}$	
	$-1-2\sqrt{3}-3+3+3\sqrt{3}=1+\sqrt{3}+p$	dM1
	$p=-2$	A1 **[3]**
M1	Substitute $\mathrm{i}\left(1+\sqrt{3}\right)$ for w and z	
dM1	Solve to $p = \ldots$	
A1	Correct value for p	
	Some solve for p first:	
M1	Obtain an expression for p in terms of w and/or z	
dM1	Substitute $\mathrm{i}\left(1+\sqrt{3}\right)$ for w and z	
A1	Correct value for p	

Question Number	Scheme	Marks
2		
(a)	$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{(r+2)^2 - r(r+3)}{r(r+1)(r+2)}$	M1
	$= \frac{r^2+4r+4-r^2-3r}{r(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)}$ *	A1* (2)
(b)	$r=1 \quad \frac{3}{1\times 2} - \frac{4}{2\times 3}$ $\qquad r=n-1 \quad \frac{n+1}{(n-1)n} - \frac{n+2}{n(n+1)}$	
	$r=2 \quad \frac{4}{2\times 3} - \frac{5}{3\times 4}$ $\qquad r=n \quad \frac{n+2}{n(n+1)} - \frac{n+3}{(n+1)(n+2)}$	M1
	$r=3 \quad \frac{5}{3\times 4} - \frac{6}{4\times 5}$	
	$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{3}{2} - \frac{n+3}{(n+1)(n+2)}$	A1
	$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{3(n+1)(n+2)-2n-6}{2(n+1)(n+2)} = \frac{n(3n+7)}{2(n+1)(n+2)}$	dM1 A1cao (4)
		[6]
(a)		
M1	Attempt a single fraction with the correct denominator (or 2 separate fractions with the correct common denominator)	
A1*	Correct result obtained with no errors in the working. Must include LHS as shown in question or LHS = ...	
(b)		
M1	Show sufficient terms to demonstrate the cancelling, min 3 at start and 1 at end or 2 at start and 2 at end. Award by implication if the correct 2 remaining terms are seen	
A1	Extract the correct 2 remaining terms	
dM1	Attempt common denominator of the form $k(n+1)(n+2)$	
A1cao	Correct result obtained. No need to show a, b and c explicitly.	

Question Number	Scheme	Marks
3		
	$x^2+x-2<\frac{1}{2}x+\frac{5}{2}$	
	$2x^2+x-9<0$	M1
	CVs $x=\frac{-1\pm\sqrt{73}}{4}$	A1
	$-x^2-x+2<\frac{1}{2}x+\frac{5}{2}$	M1
	$2x^2+3x+1>0 \quad (2x+1)(x+1)>0$	M1
	CVs $x=-\frac{1}{2},\ -1$	A1
	$\frac{-1-\sqrt{73}}{4}<x<-1, \quad -\frac{1}{2}<x<\frac{-1+\sqrt{73}}{4}$	M1A1
		[7]

	No algebra implies no marks
NB	The first 5 marks can all be awarded if equations rather than inequalities are shown
M1	Obtain and solve a 3TQ (any valid method including calculator)
A1	2 correct CVs Allow decimal equivalents (1.886..., -2.386...), min 3 sf, rounded or truncated
M1	Multiply either side by -1
M1	Obtain and solve a 3TQ (any valid method including calculator)
A1	2 correct CVs
M1	Form 2 double inequalities with their CVs. No overlap between these inequalities. Correct inequality signs required here or for final mark
A1	Correct inequalities obtained. Values must be exact, but note that 0.5 is exact. Allow “and” but not "$\cap$". May be written in set language with "$\cup$" and round brackets

Question Number	Scheme	Marks
4 (a)	$y^2 = z^{-1} \Rightarrow 2y\dfrac{dy}{dx} = -\dfrac{1}{z^2}\dfrac{dz}{dx}$ oe eg $\dfrac{dy}{dx} = -\dfrac{1}{2}z^{-\frac{3}{2}}\dfrac{dz}{dx}$	B1
	$2y\dfrac{dy}{dx} + 4y^2 = 6xy^4$	
	$-\dfrac{1}{z^2}\dfrac{dz}{dx} + \dfrac{4}{z} = \dfrac{6x}{z^2}$	M1
	$\dfrac{dz}{dx} - 4z = -6x$ *	A1 * (3)
(b)	$\text{IF} = e^{\int -4dx} = e^{-4x}$	B1
	$e^{-4x}\left(\dfrac{dz}{dx} - 4z\right) = e^{-4x} \times -6x$	
	$ze^{-4x} = -6\int xe^{-4x}dx$	M1
	$= -6\left[-\dfrac{1}{4}xe^{-4x} + \int \dfrac{1}{4}e^{-4x}dx\right]$	M1
	$= -6\left[-\dfrac{1}{4}xe^{-4x} - \dfrac{1}{16}e^{-4x}\right](+c)$ oe	A1
	$= \dfrac{3}{2}xe^{-4x} + \dfrac{3}{8}e^{-4x}(+c)$	
	$z = \dfrac{3}{2}x + \dfrac{3}{8} + ce^{4x}$ oe	A1 (5)
ALT	$\dfrac{dz}{dx} - 4z = -6x$	
	$m - 4 = 0 \Rightarrow m = 4 \Rightarrow$ CF is $z = Ae^{4x}$	B1
	PI: $z = \lambda + \mu x$	M1
	$\dfrac{dz}{dx} = \mu \Rightarrow \mu - 4(\lambda + \mu x) = -6x$	
	$4\mu = 6 \quad 4\lambda = \mu, \Rightarrow \mu = \dfrac{3}{2}, \lambda = \dfrac{3}{8}$	M1,A1
	$z = \dfrac{3}{2}x + \dfrac{3}{8} + Ae^{4x}$	A1
(c)	$y^2 = \dfrac{1}{\frac{3}{2}x + \frac{3}{8} + ce^{4x}} = \dfrac{8}{(12x + 3 + Ae^{4x})}$ oe	B1ft (1)
		[9]

Question Number	Scheme	Marks
(a)		
B1	Correct derivative seen explicitly or used	
M1	Substitutions made. Only award when an equation in x and z only is reached (if working equation I to II) or an equation in x and y is reached (if working II to I)	
A1 *	Correct result obtained with no errors in working	
(b)		
B1	Correct IF seen explicitly or used	
M1	Multiply through by their IF and integrate the LHS. Accept I for e^{-4x} on LHS only	
M1	Apply parts in the correct direction to RHS to obtain $Axe^{-4x} + B\int e^{-4x}dx$ with $A = \pm\frac{3}{2}$ and $B = \pm\frac{3}{2}$	
A1	Correct integration of RHS, constant not needed	
A1	Include the constant and treat it correctly. Answer in form $z = ...$	
ALT		
B1	Correct CF May not be seen until GS is formed	
M1	For a PI of the correct form	
M1	Differentiate their PI, substitute in the equation and extract 2 equations for the unknowns	
A1	Solve the two equations to obtain correct values for the unknowns	
A1	Correct GS obtained	
(c)		
B1ft	Any equivalent to that shown. (no need to change letter for constant if rearranged) Must start $y^2 = ...$ and must include a constant.	

Question Number	Scheme	Marks
5(a)	$-2x\dfrac{d^2y}{dx^2}+\left(2-x^2\right)\dfrac{d^3y}{dx^3}$	M1
	$+5\left(\dfrac{dy}{dx}\right)^2+5x\times 2\dfrac{dy}{dx}\dfrac{d^2y}{dx^2},\ =3\dfrac{dy}{dx}$	M1A1, B1
	$\dfrac{d^3y}{dx^3}\left(2-x^2\right)+\dfrac{d^2y}{dx^2}\left(10x\dfrac{dy}{dx}-2x\right)+5\left(\dfrac{dy}{dx}\right)^2=3\dfrac{dy}{dx}$	
	$\dfrac{d^3y}{dx^3}=\dfrac{1}{\left(2-x^2\right)}\left(2x\dfrac{d^2y}{dx^2}\left(1-5\dfrac{dy}{dx}\right)-5\left(\dfrac{dy}{dx}\right)^2+3\dfrac{dy}{dx}\right)$ *	A1* (5)
ALT 1	$\dfrac{d^2y}{dx^2}=\dfrac{3y-5x\left(\dfrac{dy}{dx}\right)^2}{\left(2-x^2\right)}$	
	$\dfrac{d^3y}{dx^3}=\dfrac{\left[3\dfrac{dy}{dx}-5\left(\dfrac{dy}{dx}\right)^2-5x\times 2\dfrac{dy}{dx}\dfrac{d^2y}{dx^2}\right]\left(2-x^2\right)-\left[3y-5x\left(\dfrac{dy}{dx}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$	M1M1A1
	$\dfrac{d^3y}{dx^3}=\dfrac{\left[3\dfrac{dy}{dx}-5\left(\dfrac{dy}{dx}\right)^2-10x\dfrac{dy}{dx}\dfrac{d^2y}{dx^2}\right]\left(2-x^2\right)+2x\left(2-x^2\right)\dfrac{d^2y}{dx^2}}{\left(2-x^2\right)^2}$	M1 (NB: B1 on ePEN)
	$\dfrac{d^3y}{dx^3}=\dfrac{1}{\left(2-x^2\right)}\left(2x\dfrac{d^2y}{dx^2}\left(1-5\dfrac{dy}{dx}\right)-5\left(\dfrac{dy}{dx}\right)^2+3\dfrac{dy}{dx}\right)$ *	A1* (5)

Question Number	Scheme	Marks
ALT 2	$\frac{d^2y}{dx^2}=\frac{3y}{(2-x^2)}-\frac{5x\left(\frac{dy}{dx}\right)^2}{(2-x^2)}$	
	$\frac{d^3y}{dx^3}=\frac{3\frac{dy}{dx}(2-x^2)-3y(-2x)}{(2-x^2)^2}$ $-\frac{\left[5\left(\frac{dy}{dx}\right)^2+5x\times 2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)-5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$	M1M1A1
	$\frac{d^3y}{dx^3}=\frac{3\frac{dy}{dx}(2-x^2)-\left((2-x^2)\frac{d^2y}{dx^2}+5x\frac{dy}{dx}\right)(-2x)}{(2-x^2)^2}$ $-\frac{\left[5\left(\frac{dy}{dx}\right)^2+5x\times 2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)-5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$	M1(B1 on ePEN)
	$\frac{d^3y}{dx^3}=\frac{1}{(2-x^2)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)$ *	A1*
(b)	$x=0 \Rightarrow 2\frac{d^2y}{dx^2}=9 \quad \frac{d^2y}{dx^2}=\frac{9}{2}$	B1
	$\frac{d^3y}{dx^3}=\frac{1}{2}\left(-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)=\frac{1}{2}\left(-5\times\frac{1}{16}+\frac{3}{4}\right)=\frac{7}{32}$	M1
	$y=3+\frac{1}{4}x+\frac{9}{2}\frac{x^2}{2!}+\frac{7}{32}\frac{x^3}{3!}$	M1
	$y=3+\frac{1}{4}x+\frac{9}{4}x^2+\frac{7}{192}x^3$	A1 (4)
		[9]

Question Number	Scheme	Marks
(a)		
M1	Differentiate $\left(2-x^2\right)\frac{d^2y}{dx^2}$ using product rule	
M1	Differentiate $5x\left(\frac{dy}{dx}\right)^2$ using product and chain rule	
A1	Correct derivative of $5x\left(\frac{dy}{dx}\right)^2$	
B1	Correct derivative of $3y$	
A1*	Correct result obtained from fully correct working	
ALT 1	*Rearrange and use quotient rule*	
M1	Use the quotient rule. Denominator must be $\left(2-x^2\right)^2$ and numerator to be the difference of 2 terms	
M1	Differentiate $\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\frac{dy}{dx}$	
A1*	Correct result obtained from fully correct working	
ALT 2	*Rearrange, separate into 2 fractions and then use quotient* rule	
M1	Use the quotient rule on both fractions. Denominators must be $\left(2-x^2\right)^2$ and numerator of each to be the difference of 2 terms	
M1	Differentiate $3y$ using the chain rule **and** differentiate $5x\left(\frac{dy}{dx}\right)^2$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\frac{dy}{dx}$	
A1*	Correct result obtained from fully correct working	
(b)		
B1	Correct value of $\frac{d^2y}{dx^2}$	
M1	Use the given result from (a) to obtain a value for $\frac{d^3y}{dx^3}$	
M1	Taylor's series formed using their values for the derivatives (accept 2! or 2 and 3! or 6)	
A1	Correct series, must start (or end) $y = \ldots$ but accept $f(x)$ provided $y = f(x)$ defined somewhere	

Question Number	Scheme	Marks
6(a)	$m^2+2m+5=0 \Rightarrow m=-1\pm 2i$	M1
	C F: $y=\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)$	
	OR $y=\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)$ or $y=P\mathrm{e}^{(-1+2\mathrm{i})x}+Q\mathrm{e}^{(-1-2\mathrm{i})x}$	A1
	PI: $y=a\cos x+b\sin x$	B1
	$y'=-a\sin x+b\cos x \qquad y''=-a\cos x-b\sin x$	
	$-a\cos x-b\sin x-2a\sin x+2b\cos x+5a\cos x+5b\sin x=6\cos x$	M1
	$-b-2a+5b=0 \qquad -a+2b+5a=6$	M1
	$a=\frac{6}{5} \quad b=\frac{3}{5}$	A1
	GS: $y=$ their CF $+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1ft (7)
(b)	$x=0, y=0 \;\; 0=A+\frac{6}{5} \;\Rightarrow\; A=-\frac{6}{5}$	M1
	$y'=-\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)+\mathrm{e}^{-x}\left(-2A\sin 2x+2B\cos 2x\right)$	
	$-\frac{6}{5}\sin x+\frac{3}{5}\cos x$	M1A1ft
	$x=0 \;\; \frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 0=+\frac{6}{5}+2B+\frac{3}{5} \;\Rightarrow\; B=-\frac{9}{10}$	dM1
	PS: $y=\mathrm{e}^{-x}\left(-\frac{6}{5}\cos 2x-\frac{9}{10}\sin 2x\right)+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1 (5)
		[12]
ALT	$y=\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	
	$x=0 \;\; y=0 \;\; 0=P+Q+\frac{6}{5}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-x}\left(2\mathrm{i}P\mathrm{e}^{\mathrm{i}2x}-2\mathrm{i}Q\mathrm{e}^{-\mathrm{i}2x}\right)-\mathrm{e}^{-x}\left(P\mathrm{e}^{\mathrm{i}2x}+Q\mathrm{e}^{-\mathrm{i}2x}\right)-\frac{6}{5}\sin x+\frac{3}{5}\cos x$	M1A1ft
	$0=2\mathrm{i}P-2\mathrm{i}Q+\frac{9}{5}$	
	$P+Q=-\frac{6}{5} \qquad P-Q=\frac{9}{10}\mathrm{i}$	
	$P=\frac{1}{2}\left(-\frac{6}{5}+\frac{9}{10}\mathrm{i}\right) \qquad Q=\frac{1}{2}\left(-\frac{6}{5}-\frac{9}{10}\mathrm{i}\right)$	dM1
	PS: $y=\frac{1}{2}\mathrm{e}^{-x}\left(-\frac{6}{5}+\frac{9}{10}\mathrm{i}\right)\mathrm{e}^{2\mathrm{i}x}+\frac{1}{2}\mathrm{e}^{-x}\left(-\frac{6}{5}-\frac{9}{10}\mathrm{i}\right)\mathrm{e}^{-2\mathrm{i}x}+\frac{6}{5}\cos x+\frac{3}{5}\sin x$	A1 (5)

Question Number	Scheme	Marks
(a)		
M1	Form and solve the auxiliary equation	
A1	Correct CF, either form (Often not seen until GS stated)	
B1	Correct form for the PI	
M1	Differentiate twice and sub in the original equation	
M1	Obtain a pair of simultaneous equations and attempt to solve	
A1	Correct values for both unknowns	
A1ft	Form the GS. Must start $y = \ldots$ Follow through their CF (writing CF scores A0) Must have scored a minimum of 2 of the M marks	
(b)		
	For CF $y = e^{-x}\left(A\cos 2x + B\sin 2x\right)$	
M1	Sub $x = 0,\ y = 0$ in their GS and obtain a value for A	
M1	Differentiate their GS Product rule must be used	
A1ft	Correct differentiation of their GS provided this has 4 terms	
dM1	Sub $x = 0,\ \frac{dy}{dx} = 0$ and their A and obtain a value for B Depends on both previous M marks	
A1	Fully correct PS. Must start $y = \ldots$	
ALT(b)		
	For CF $y = e^{-x}\left(Pe^{i2x} + Qe^{-i2x}\right)$ or $y = Pe^{(-1+2i)x} + Qe^{(-1-2i)x}$	
M1	Sub $x = 0,\ y = 0$ in their GS and obtain an equation in P and Q	
M1	Differentiate their GS Product rule must be used if $y = e^{-x}\left(Pe^{i2x} + Qe^{-i2x}\right)$ used	
A1ft	Correct differentiation of their GS	
dM1	Sub $x = 0,\ \frac{dy}{dx} = 0$ to obtain a second equation and solve the pair of equations The solution must allow for P and Q to be complex	
A1	Fully correct PS. Must start $y = \ldots$	

Question Number	Scheme	Marks
7 **(a)**	$x = r\cos\theta = 3\sin 2\theta\cos\theta$	B1
	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 6\cos 2\theta\cos\theta - 3\sin 2\theta\sin\theta = 0$	M1
	$2\cos\theta\left(\cos^2\theta - 2\sin^2\theta\right) = 0$	M1
ALT	For the 2 M marks:	
	$x = 6\sin\theta\cos^2\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 6\cos^3\theta - 12\sin^2\theta\cos\theta = 0$	
	$\tan\phi = \dfrac{1}{\sqrt{2}}$ *	A1* (4)
(b)	$\tan\phi = \dfrac{1}{\sqrt{2}} \Rightarrow \sin\phi = \dfrac{1}{\sqrt{3}},\ \cos\phi = \dfrac{\sqrt{2}}{\sqrt{3}}$	M1
	$R = 3\times 2\times\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{2}}{\sqrt{3}} = 2\sqrt{2}$	A1 (2)
(c)	Area of sector $= \dfrac{1}{2}\int r^2\,\mathrm{d}\theta = \dfrac{9}{2}\int \sin^2 2\theta\,\mathrm{d}\theta$	M1
	$= \dfrac{9}{2}\int_0^{\arctan\left(\frac{1}{\sqrt{2}}\right)} \dfrac{1}{2}\left(1-\cos 4\theta\right)\mathrm{d}\theta$	M1
	$= \dfrac{9}{2}\left[\dfrac{1}{2}\left(\theta - \dfrac{1}{4}\sin 4\theta\right)\right]_0^{\arctan\frac{1}{\sqrt{2}}}$	M1A1
	$= \dfrac{9}{4}\left[\arctan\dfrac{1}{\sqrt{2}} - \dfrac{1}{4}\sin 4\left(\arctan\dfrac{1}{\sqrt{2}}\right) - 0\right]$	dM1
	$\sin 4\phi = 2\sin 2\phi\cos 2\phi = 4\sin\phi\cos\phi\left(2\cos^2\phi - 1\right)$ $= 4\times\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{2}}{\sqrt{3}}\left(2\times\dfrac{2}{3}-1\right) = \dfrac{4\sqrt{2}}{9}$	M1
	Area of sector $= \dfrac{9}{4}\left(\arctan\dfrac{1}{\sqrt{2}} - \dfrac{1}{4}\times\dfrac{4\sqrt{2}}{9}\right) = \dfrac{9}{4}\arctan\dfrac{1}{\sqrt{2}} - \dfrac{\sqrt{2}}{4}$	A1 (7)
		[13]

Question Number	Scheme	Marks
(a)		
B1	State $x = (r\cos\theta) = 3\sin 2\theta\cos 2\theta$ May be given by implication	
M1	Attempt to differentiate $x = r\cos\theta$ or $x = r\sin\theta$ Product rule must be used	
M1	Use a correct double angle formula **and** equate the derivative of $r\cos\theta$ to 0	
ALT	**M1** Attempt the differentiation of $x = r\cos\theta$ or $x = r\sin\theta$ using the product rule (after using a double angle formula) **M1** Use a correct double angle formula **and** equate the derivative of $r\cos\theta$ to 0	
A1*	Complete to the given answer and no extras with no errors in the working. Accept θ or ϕ All values seen must be exact	
(b)		
M1	Attempt exact values for $\sin\theta$ and $\cos\theta$ and use these to obtain a value for *R*. Values for $\sin\theta$ and/or $\cos\theta$ may have been seen in (a)	
A1	A correct, exact value for *R*, as shown or any equivalent. Award M1A1 for a correct exact answer	
(c)		
M1	Use of Area $=\frac{1}{2}\int r^2 \mathrm{d}\theta$ Limits not needed (ignore any shown)	
M1	Use the double angle formula to obtain $k\int \frac{1}{2}(1\pm\cos 4\theta)\mathrm{d}\theta$ Ignore any limits given This is NOT dependent NB: There are other, lengthy, methods of reaching this point	
M1	Attempt the integration $\cos 4\theta \to \pm\frac{1}{4}\sin 4\theta$ (Not dependent)	
A1	Correct integration of $1-\cos 4\theta$	
dM1	Correct use of correct limits. Depends on second and third M marks 0 at lower limit need not be shown	
M1	Attempt an exact numerical value for $\sin 4\left(\arctan\frac{1}{\sqrt{2}}\right)$	
A1	Correct final answer. Award M1A1 for a correct exact final answer	

Question Number	Scheme	Marks
8(a)	$z^n = e^{in\theta} = \cos n\theta + i\sin n\theta$	
	$\frac{1}{z^n} = e^{-in\theta} = \cos(-n\theta) + i\sin(-n\theta) = \cos n\theta - i\sin n\theta$	
	$z^n + \frac{1}{z^n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta = 2\cos n\theta$ *	M1A1cso (2)
(b)	$\left(z+\frac{1}{z}\right)^6 = z^6 + 6z^5 \times \frac{1}{z} + \frac{6\times5}{2!}z^4 \times \frac{1}{z^2} + \frac{6\times5\times4}{3!}z^3 \times \frac{1}{z^3}$ $+ \frac{6\times5\times4\times3}{4!}z^2 \times \frac{1}{z^4} + \frac{6\times5\times4\times3\times2}{5!}z \times \frac{1}{z^5} + \frac{1}{z^6}$	M1A1
	$(2\cos\theta)^6 = z^6 + 6z^4 + 15z^2 + 20 + 15\times\frac{1}{z^2} + 6\times\frac{1}{z^4} + \frac{1}{z^6}$	
	$64\cos^6\theta = z^6 + \frac{1}{z^6} + 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) + 20$	M1
	$64\cos^6\theta = 2\cos 6\theta + 6\times2\cos 4\theta + 15\times2\cos 2\theta + 20$	M1
	$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$ *	A1* (5)
(c)	$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 = 10$	
	$32\cos^6\theta = 10$	M1A1
	$\cos\theta = \pm\sqrt[6]{\frac{5}{16}}$	
	$\theta = 0.6027...,\ 2.5388... \qquad \theta = 0.603,\ 2.54$	M1A1 (4)
(d)	$\int_0^{\frac{\pi}{3}} (32\cos^6\theta - 4\cos^2\theta)\,d\theta$	
	$= \int_0^{\frac{\pi}{3}} (\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 - 4\cos^2\theta)\,d\theta$	
	$= \int_0^{\frac{\pi}{3}} (\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 - 2 - 2\cos 2\theta)\,d\theta$	M1
	$= \left[\frac{1}{6}\sin 6\theta + \frac{3}{2}\sin 4\theta + \frac{13}{2}\sin 2\theta + 8\theta\right]_0^{\frac{\pi}{3}}$	M1A1
	$= (0) + \frac{3}{2}\left(-\frac{\sqrt{3}}{2}\right) + \frac{13}{2}\times\frac{\sqrt{3}}{2} + \frac{8\pi}{3} \quad (-0)$	dM1
	$= \frac{5\sqrt{3}}{2} + \frac{8\pi}{3}$ oe	A1 (5)
		[16]

Question Number	Scheme	Marks
(a)		
M1	Attempt to obtain $z^n + \frac{1}{z^n}$	
A1cso	Reach the given result with clear working and no errors Must see $\cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ changed to $\cos n\theta - \mathrm{i}\sin n\theta$ (ie both included)	
(b)	*The first 3 marks apply to the binomial expansion only*	
M1	Apply the binomial expansion to $\left(z + \frac{1}{z}\right)^6$ Coefficients must be numerical (ie ${}^nC_r$ is not acceptable). The expansion must have 7 terms with at least 4 correct	
A1	Correct expansion, terms need not be simplified	
M1	Simplify the coefficients and pair the appropriate terms on RHS (At least 2 pairs must be correct)	
M1	Use the result from (a) throughout. Must include 2^6 or 64 now	
A1*	Obtain the given result with no errors in the working	
(c)		
M1	Use the result from (b) to simplify the given equation	
A1	Reach $32\cos^6\theta = 10$ oe	
M1	Solve to obtain at least one correct value for θ, in radians and in the given range, 3 sf or better	
A1	2 correct values, and no extras, in radians and in the given range. Must be 3 sf here Ignore extras outside the range	
(d)		
M1	Use the result in (b) to change $\cos^6\theta$ to a sum of multiple angles ready for integration and use $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$ on $\cos^2\theta$ Limits not needed, ignore any shown	
M1	Integrate their expression to obtain an expression containing terms in $\sin 6\theta, \sin 4\theta, \sin 2\theta$ and θ Limits not needed	
A1	Correct integration Limits not needed	
dM1	Substitute limit pi/3. Depends second M mark	
A1	Correct, exact, answer (any equivalent to that shown). AwardM1A1 for a correct final answer following fully correct working. There are other ways to integrate the function in (d), eg parts on one or both of the powers of $\cos\theta$, using $\cos^6\theta = \left(\cos^2\theta\right)^3 = \frac{1}{8}(1 + \cos 2\theta)^3 = ...$ If in doubt about the marking of alternative methods which are not completely correct, send to review	